Handling Delayed and Out of Sequence Measurements in Chemical Plants using the Extended Kalman Filter based Data Fusion Technique

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Abstract— Sensor data can be fused with data acquired through laboratory analysis or from costly real time sensors which may offer data after a time delay. Because of these operations we may come across out of sequence measurement (OOSM) data. In this work, fixed point smoothing is used to deal with the OOSM problem. An extended Kalman filter (EKF) is utilized to fuse data. The developed algorithm is tested on a distillation column and a continuous stirred tank reactor (CSTR).

Index Terms— data fusion, sensor fusion, out of sequence measurement, extended Kalman filter, fixed point smoothing, distillation column, continuous stirred tank reactor.

1 INTRODUCTION

In chemical processes, the foremost problem for effective quality control is the shortage of real-time measurements of the major variables, due to time delays and lengthy and irregular sampling times. Instead, predictive models can be designed based on the stored measurement data and/or mechanism information. The model gives the relationship between obtainable process variables and quality variables to provide fast estimations of the difficult-to-measure quality variables.

There are numerous modeling methods [1]. Every modeling method unavoidably has limitations, owing to their internal operation mechanism and the variety of diverse industrial processes. In addition, a chemical process is typically complex and time-varying, which also increases the effort required for accurate model estimation.

To efficiently supplement the model estimation with several instrumental observations under noises, data fusion method is a decent choice. By assimilating multi-source observations, data fusion process could extract the useful information and resist disturbances [2]. Kalman filter is one of the most extensively used data fusion methods. It is an optimal observer based on the information of process and related noises. Kalman filter has the benefit of multi-source data fusion, dynamic execution and noise resisting. Meanwhile, it is suitable for multi-rate and delayed measurement systems. The Kalman filter method is a recursive algebraic algorithm, which is also appropriate for online applications [3].

The idea of including delayed measurements within a Kalman filter structure has been well documented in the literature. In tracking and navigation systems substantial time delays get introduced due to the involvedness of computation or network delay from multiple sensors sending data to the estimator. The sampling times are of the order of milliseconds and techniques anticipated in literature to deal with delays [4] mainly depend on fusing the information from the delayed measurement directly into the filter when it arrives.

In chemical process systems, the problem of dealing with delayed measurements typically occurs as a result of delays in measurement of certain quality variables. These measurements (called primary measurements) are sampled rarely and are accessible with a delay, while other variables are measured regularly and the measurements are obtainable instantaneously (secondary measurements). For example, in distillation columns the distillate and bottoms compositions are frequently required to be analyzed in a laboratory. Because the employment and up keeping of online analyzers is impracticable due to economic considerations, whereas secondary measurements such as tray temperatures are acquired regularly without any time delay. The primary variables are thus inferred from the temperature measurements, which might be inaccurate owing to model errors (plant-model mismatch), sensor bias or un-modeled disturbances. In such cases the primary measurements, which are available with a delay because of offline assesses, is required to be combined into the estimator. Time delays in multi-rate estimation have frequently been dealt with in literature for process systems using appropriate augmentation of the states. Gudi et al. [5] employed multi-rate state estimation techniques with delayed measurements for a fermentation process in a bioreactor. Tatiraju et al. [6] studied estimation techniques for a polymerization reactor in the company of delayed measurements of the molecular weight distribution. Mutha et al. [7] suggested a technique for multi-rate state estimation in a polymerization reactor that compensates for the slow measurements by repeatedly using the available slow measurements. Amirthalingam et al. [8] augmented the state with past measurements to deal with delays in the primary measurement for an identified linear

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ISSN 2229-5518

model. The state augmentation technique is employed in the chemical and biochemical processes because the state-space design is maintained, which simplifies the extension of delay-managing methods to other techniques, such as moving horizon estimation (MHE) [9], nonlinear dynamic data reconciliation (NDDR) [10], unscented Kalman filter (UKF), particle filters, etc. In many examples, the time delay for primary measurement was of the order of a few sampling periods (usually one or two samples) was assumed to be constant and known in advance. Extension to indeterminate delays, while abstractly clear, has not received enough consideration. When time delay is large and/or indefinite, it is required to use techniques that can deal with uneven and time-varying delays competently. The aim of this paper is to offer an analysis and to evaluate through simulation studies a method to cope with measurement delays in multi-rate estimation using extended Kalman filter (EKF) for process systems. Extension of fixed point smoothing Kalman filter to manage out of sequence measurements as well as simultaneously receiving multiple measurements are discussed [11].

This paper is organized as follows. Section 2 defines the basic system and filter equations. In Section 3 the techniques for state-vector fusion and measurement fusion are presented. Section 4 discloses the out of sequence measurement problem. The algorithm coping with measurement delays within the EKF structure is explained in section 5. The distillation column and CSTR case studies are explained in Section 6. Results and discussions are given in Section 7. Finally, Section 8 sums up the conclusions of this work.

2 Nonlinear Model Estimator

The system in this study is described by a state space model. At time k the state is represented as X_k, and the input is designated as v_k. The state space model of the system is given as:

$$X_{k+1} = f\left(X_k, u_k\right) + v_k \tag{1}$$

where v_k is the Gaussian process noise. At time k the process measurements are given by:

$$Y_k = g\left(X_k, u_k\right) + w_k \tag{2}$$

where w_k is the Gaussian measurement noise. The two noise sources v_k and w_k are assumed to be normally distributed white noise sequences with zero mean and covariance matrices Q and R, respectively.

To resolve the state estimation problem an extended Kalman filter (EKF) is used [12]. This procedure involves two steps: the time update step and the measurement update step. In the time update step, (1) is used to update the states and covariances as:

$$\hat{X}_{k+1|k} = F_k \hat{X}_{k|k} \tag{3}$$

$$P_{k+1|k} = F_k P_{k|k} F^T + Q \tag{4}$$

where $\hat{X}_{k|k}$ should be taken as the estimate of X at time k given measurements up to time k and F_k is the linearization of (1) about $\hat{X}_{k|k-1}$. When a measurement is obtained, (5) is used to update the mean of the state estimate. To update the covariance estimate, the measurement model is linearized at $\hat{X}_{k|k-1}$, i.e., the estimate of the previous time update step.

The Kalman filter equations for the measurement update are:

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k\left(y_k - \mathrm{g}\left(\hat{X}_{k|k-1}\right)\right) \tag{5}$$

$$P_{k|k} = \left(I - K_k G_k\right) P_{k|k-1} \tag{6}$$

where G_k is the linearization of (2) around $\hat{X}_{k|k-1}$, and the Kalman gain is:

$$K_k = P_{k|k-1} G_k^T \left(G_k P_{k|k-1} G_k^T - R\right)^{-1} \tag{7}$$

3 Measurement Fusion Model

For simplicity, consider that the sensors' sample rates are alike and the dynamics of the system is given by (1). The measurements corresponding to the two sensors are:

$$Y_k^m = G_k^m X_k + w_k^m, m = 1,2 \tag{8}$$

where Y_k^m is the measurement of the sensor m at time k and the measurement noise sequences w_k^m are zero-mean, white, with covariance R_k^m, and the sensors are mutually independent, i.e.,

$$E\left[w_k^m\right] = 0; E\left[w_k^m w_l^{mT}\right] = R_k^m \delta_{kl} \tag{9}$$

$$E\left[w_k^1 w_l^{2T}\right] = E\left[w_k^2 w_l^{1T}\right] = 0 \tag{10}$$

Fusion of these tracks can now occur at either the state vector or measurement vector level.

The fusion merely merges the measurements into an augmented observation vector [13]. The measurement vectors Y_k^1 and Y_k^2 from the two (or more) sensors are merged into a new augmented measurement vector given by:

$$Y_k = \left[\left(Y_k^1\right)^T \left(Y_k^2\right)^T\right]^T \tag{11}$$

Denote $G_k = \left[\left(G_k^1\right)^T \left(G_k^2\right)^T\right]^T$ and $w_k = \left[\left(w_k^1\right)^T \left(w_k^2\right)^T\right]^T$

Then from (8) and (11) a new measurement equation is given by:

$$Y_k = G_k X_k + w_k \tag{12}$$

Based on the presumed statistical independence of the two sensors the covariance matrix R for the merged measurement noise w_k is given as:

$$R = \begin{pmatrix} R^1 & 0 \\ 0 & R^2 \end{pmatrix} \tag{13}$$

Consequently, the estimate, $\hat{X}_{k|k}$, of the state vector can be determined by the conventional Kalman filter. The measurement fusion procedure is shown in Fig. 1.

4 Out of Sequence Measurements Problem

Fig. 2 shows an illustration of a process with irregular and delayed measurements [12], [16]. Here we assume that the sensor A has the smallest delay time. Its delay time is

considered to be equal to zero.

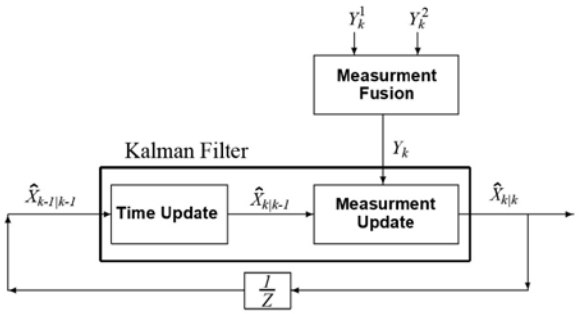

Fig. 1. The measurement fusion process.

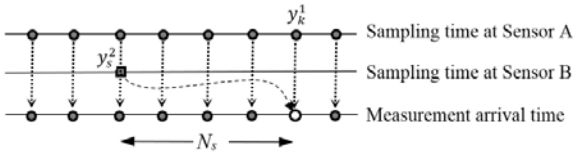

Fig. 2. The scenario of centralized fusion where the OOS phenomenon occurs.

In the first sampling time line (at the top) the circles represent the sampling times by the sensor A. The square at the middle sampling time line represents a sample from the sensor B. The sensor B measurement arrives with a delay of N_S time intervals. The sample that is generated by the sensor A at the time step k is denoted as y_k^1 and the sample from sensor B at the time step s is denoted as y_s^2. Consider the time step k is represented with a hollow circle (the time line at the bottom) in Fig. 2. At this time step the sensor A measurement (y_k^1) and sensor B measurement (y_s^2) arrive in the same computational instant. However, the sensor B measurement belongs to the N_S preceding sampling time. The measurement from the sensor with a longer delay time (i.e., sensor B) is considered as an OOS measurement.

The output of the sensor A is given as:

$$Y_k^1 = g^1(X_k) + w_k^1 \tag{14}$$

Since the sensor B measurement arriving at time k is related to the state at time s, the output of the primary measurement is given by:

$$Y_k^2 = g^2(X_s) + w_s^2 \tag{15}$$

We end this section by noting that the linearized form of the outputs given by (14) and (15) are:

$$G_k^i = \left.\frac{\partial g^i(X)}{\partial X}\right|_{\hat{X}_{k|k-1}}, \quad i = \{1, 2\} \tag{16}$$

Measurement delays cause problems in multi-rate sensor estimations in process systems. This necessitates the use of alternate methods that can take into consideration the out of sequence measurements.

5 Handling Out of Sequence Measurements

To solve the problem of out of sequence measurements we employ the fixed point smoothing method. Since the delayed measurement is a function of the state X_s, only the information of this state is required to be retained until the measurement is performed. The aim in fixed point smoothing is to attain a priori state estimates of X_s at times s+1, s+2, …, k,…. We will use the notation $X_{s|k}$ to refer to the estimate of X_s attained by using all the measurements up to and including the time step (k-1).

That is, $\hat{X}_{s|k}$ can be considered as the a priori estimate of X_s at time k:

$$\hat{X}_{s|k} = E\left(X_S | Y_1, \ldots, Y_{k-1}\right), k \geq s \tag{17}$$

Thus, when the primary variable is sampled, we will assume a new state variable X'. This new state variable will be initialized as $X'(s) = X(s)$. This idea is depicted in Fig. 3.

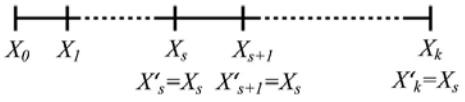

Fig. 3. This illustrates the idea that is used to obtain the fixed-point smoother.

Augmenting the dynamics of our newly defined state X' to the original system results in the following:

$$\begin{bmatrix} X_{k+1} \\ X'_{k+1} \end{bmatrix} = \begin{bmatrix} F_k & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} X_k \\ X'_k \end{bmatrix} + \begin{bmatrix} I \\ 0 \end{bmatrix} v_{k-1} \tag{18}$$

$$Y(k) = \begin{bmatrix} G_k & 0 \end{bmatrix} \begin{bmatrix} X(k) \\ X'(k) \end{bmatrix} + w_k \tag{19}$$

The estimation is performed by the following equations:

$$\hat{X}_{k+1} = F_k \hat{X}_k + K_k \left(y_k - g\left(\hat{X}_k\right)\right) \tag{20}$$

$$\hat{X}'_{k+1} = \hat{X}'_k + \lambda_k \left(y_k - g\left(\hat{X}'_k\right)\right) \tag{21}$$

$$K_k = P_k G_k^T \left(G_k P_k G_k^T + R\right)^{-1} \tag{22}$$

$$\lambda_k = \Sigma_k G_k^T \left(G_k P_k G_k^T + R\right)^{-1} \tag{23}$$

$$P_{k+1} = F_k P_k \left(F_k - K'_k G_k\right)^{-1} + Q \tag{24}$$

$$\Pi_{k+1} = \Pi_k - \Sigma_k G_k^T \lambda_k^T \tag{25}$$

$$\Sigma_{k+1} = \Sigma_k \left(F_k - K'_k G_k\right)^T \tag{26}$$

where K_k is the standard Kalman filter gain of the estimate of X, λ_k is the Kalman filter gain of the smoothed estimate of X at the time step k, Π_k is the state error covariance of the smoothed estimate of X at the time step k and Σ_k is the cross covariance between P_k and Π_k.

The Kalman filter will consequently update both the current state $\hat{X}_k$ and the lagged state $\hat{X}'_s$ during the measurement delay. When the delayed measurement is received at the sampling point (s +N_s), it is fused with the

smoothed estimate of $\hat{X}_s'$. The state space matrix K' (the augmented Kalman filter gain) defined below by (27) is substituted for K_k used in (3) through (7). Similarly, the state space matrix P' (the augmented state error covariance matrix) defined by (28) is substituted for P_k used in (3) through (7).

$$K' = \begin{bmatrix} K'_k \\ \lambda_k \end{bmatrix} \tag{27}$$

$$P' = \begin{bmatrix} P_{k+1} & \Sigma^T_{k+1} \\ \Sigma_{k+1} & \Pi_{k+1} \end{bmatrix} \tag{28}$$

After the primary measurement is perfectly fused at the sampling point ($s + N_s$), the lagged state $\hat{X}_s'$ is not needed any more. This technique can be applied to the case of indeterminate and time varying delays.

6 Description of Processes

6.1 The Binary Distillation Column

Distillation columns are usually good testing plants for nonlinear model identification. Hence, we consider a distillation column with 30 trays for the separation of a binary mixture based on the proposed model by Hahn and Edgar [15]. The feed stream is introduced at the middle of the column on stage 17. Fig. 4 shows the schematic of this column.

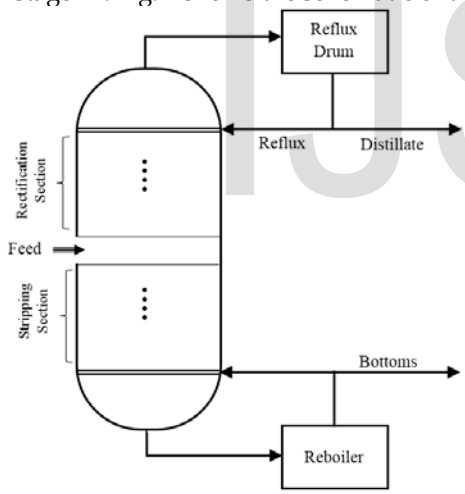

Fig. 4. A schematic of the binary distillation column.

The purity of the distillate is measured. The equations for this simulation are:

For the reflux drum (i = 1):

$$\frac{dx_{A,i}}{dt} = \frac{1}{A_{cond}} V_f \left(y_{A,2} - x_{A,1} \right) \tag{29}$$

where A_{cond} is the total molar holdup in the condenser, V_f is the vapor flow rate in the column (mol.min^{-1}), $x_{A,i}$ is the liquid composition of component A on the i^{th} stage and $y_{A,i}$ is the vapor composition of component A on the i^{th} stage.

For the trays in the rectification section (i = 2… 16):

$$\frac{dx_{A,i}}{dt} = \frac{1}{A_{Tray}} \left[L_1 \left(x_{A,i-1} - x_{A,i} \right) - V_f \left(y_{A,i} - y_{A,i+1} \right) \right] \tag{30}$$

where A_{tray} is the total molar holdup on each tray and L_1 is the flow rate of the liquid in the rectification section (mol.min^{-1}).

For the feed tray:

$$\frac{dx_{A,17}}{dt} = \frac{1}{A_{Tray}} \left[F_f x_{A,Feed} + \left(L_1 x_{A,16} - L_2 x_{A,17} \right) - V_f \left(y_{A,17} - y_{A,18} \right) \right] \tag{31}$$

where F_f is the feed flow rate (mol.min^{-1}), $x_{A,feed}$ is the feed composition of component A and L_2 is the flow rate of the liquid in the stripping section.

For the trays in the stripping section (i = 18… 31):

$$\frac{dx_{A,i}}{dt} = \frac{1}{A_{Tray}} \left[L_2 \left(x_{A,i-1} - x_{A,i} \right) - V \left(y_{A,i} - y_{A,i+1} \right) \right] \tag{32}$$

For the reboiler:

$$\frac{dx_{A,32}}{dt} = \frac{1}{A_{Reboiler}} \left[L_2 \left(x_{A,31} - \left(F_f - D_f \right) x_{A,32} - V y_{A,32} \right) \right] \tag{33}$$

where $A_{reboiler}$ is the total molar holdup of the reboiler and D_f is the distillate flow rate (mol.min^{-1}).

Further equations:

$$V_f = L_1 + D_f \tag{34}$$

$$L_2 = F_f + L_1 \tag{35}$$

$$R_r = L_1 / D_f \tag{36}$$

where R_r is the reflux ratio.

$$\alpha_{A,B} = \frac{y_A \left(1 - x_A \right)}{\left(1 - y_A \right) x_A} \tag{37}$$

where $\alpha_{A,B}$ is the relative volatility.

In this work, values assigned to some of the variables used in the above equations are defined in Table 1.

6.2 The Continuous Stirred Tank Reactor

A CSTR with a simple exothermal reaction $A \rightarrow B$ is shown in Fig. 5. For the development of a mathematical model for this process, the following assumptions are made: neglect the heat capacity of the inner walls of the reactor, constant density and specific heat capacity of liquid, constant reactor volume, constant overall heat transfer coefficient, and equal input and output volumetric flow rates. As the reactor is well-mixed, the outlet stream concentration and temperature are identical with those in the tank. A mass balance of the component A can be expressed as:

$$V \frac{dc_A}{dt} = q c_{AV} - q c_A - V r \left(c_A, T \right) \tag{38}$$

where V is the reactor volume (m^3), c_A is the molar concentration of A in the outlet stream (kmol.m^{-3}), q is the volumetric flow rate of reaction mixture (m^3.min^{-1}), c_{AV} is the molar concentration of A in the inlet stream (kmol.m^{-3}), r is the rate of reaction, and T is the temperature of reaction mixture. The rate of reaction is a function of concentration and temperature (Arrhenius law) and is given as:

$$r(c_A, \mathrm{T}) = k_c c_A = k_{c0} \exp(-E / (R_e T))\, c_A \quad (39)$$

TABLE 1

MODEL PARAMETERS FOR DISTILLATION COLUMN

Variable	Value
Fresh feed (F_f)	0.4 mol.min^{-1}
Distillate (D_f)	0.2 mol.min^{-1}
Liquid in the Rectification Section (L_1)	0.54 mol.min^{-1}
Vapor in the column (V_f)	0.74 mol.min^{-1}
Liquid in the Stripping Section (L_2)	0.94 mol.min^{-1}
Total molar hold up in codenser A_{cond}	0.25 mol.min^{-1}
Total molar hold up on each tray A_{tray}	0.5 mol.min^{-1}
Total molar hold up in reboiler A_{reboil}	1 mol.min^{-1}
Total Pressure in Column	101000 pa
Reflux ratio (R_r)	2.7
Mole fraction of Fresh feed ($x_{A,feed}$)	0.5
Relative volatility ($\alpha_{A,B}$)	1.6

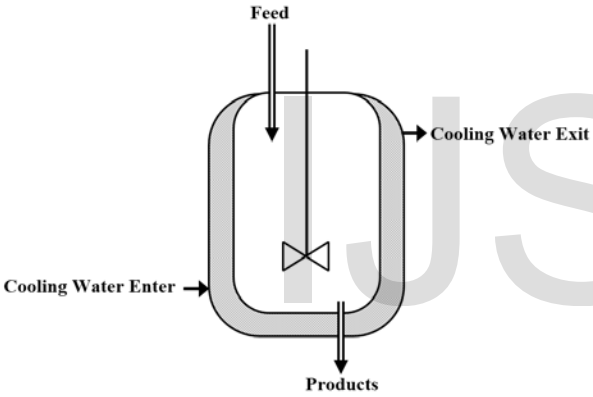

Fig. 5 . A non-isothermal CSTR.

where k_{c0} is the frequency factor, E is the activation energy (kJ.kmol^{-1}) and R_e is the Reynolds number (kJ.kmol^{-1}.K^{-1})

The heat balance gives the equation:

$$\frac{V \rho c_p dT}{dt} = q \rho c_p T_V - q \rho c_p T - \alpha F_h (T - T_c) + V (-\Delta H) r(c_A, T) \quad (40)$$

where ρ is the liquid density (kg.m^{-3}), c_p is the liquid specific heat capacity (kJ.kg^{-1}.K^{-1}), T_v is the temperature in the inlet stream (°K), α is the overall heat transfer coefficient (kJ.m^{-2}.min^{-1}.K^{-1}), F_h is the heat transfer area (m^2), Tc is the cooling temperature(°K), ΔH is the heat of reaction (kJ.kmol^{-1}).

Equations (39) and (40) express the nonlinear system model of the CSTR. Parameter values of the reaction and reactor are shown in Table 2.

7 SIMULATION RESULTS

In this section the performance of the method described in section 5 (EKF2) is compared with standard EKF described in section 2 (EKF1) using the distillation column and CSTR case studies described in section 6.

TABLE 2

PARAMETERS OF THE REACTION AND THE REACTOR

Variable	Unit	Value
C_{AVS}	kmol.m^{-3}	1.2
C_P	kJ.kg^{-1}.k^{-1}	4.05
E	kJ.kmol^{-1}	107280
F_h	m^2	6.08
k_0	min^{-1}	7.93e15
V	m^3	1.7
ΔH	kJ.kmol^{-1}	-150000
q	m^3.min^{-1}	0.2
R_e	kJ.kmol^{-1}.k^{-1}	8.314
α	Kj.m^{-2}.min^{-1}.k^{-1}	41.2
T_C	°K	318
T_V	°K	313
ρ	Kg.m^{-3}	998

7.1 Simulation Results of the Distillation Column

Details of the model derivation and description are given in section 6.1. The related equations are expressed by the following standard nonlinear state space model form:

$$\frac{dX(t)}{dt} = f(X(t), U(t)) \quad (41)$$

$$Y^1(t) = g^1(X(t)) \quad (42)$$

$$Y^2(t) = g^2(X(t)) \quad (43)$$

where $X(t)$ is the liquid mole fraction of component A on all 30 trays, the reflux drum and the reboiler; $U(t)$ is the reflux ratio (R_r); $Y^1(t)$ is the temperature of reboiler and reflux drum; $Y^2(t)$ is the liquid mole fraction of the component A in the reboiler and reflux drum.

The noise variances used are as follows. The measurement error covariances for the frequent temperature measurements, R^1=10^{-3} and for the delayed concentration measurements, R^2=10^{-8}; and the process noise, Q=10^{-6}. The EKF1 is started with an initial estimation error P_0 =10^{-5}. The process is initially at an unsteady state and the estimator is given an incorrect starting guess. Both the EKF1 method (uses only the secondary measurements) and EKF2 method (uses the secondary and the delayed primary measurements) are considered. The estimator performance is tested in the presence of the state and measurement noises.

In the EKF1 method, the secondary temperature measurements are available at every sampling instant. In the EKF2 method, the samples for the primary composition are taken at a regular interval of 12 min, starting from the time step k=1. The delay T_D in the measurement arrival is equal to 8 min.

The performance of the estimators in the presence of noise is shown in Figs. 6 and 7. In these figures, the solid lines represent the plant (actual values), the dashed lines represent the EKF1

method estimation responses and the doted lines represent the EKF2 method estimation responses. Fig. 6 shows the reflux drum liquid mole fraction of the component A and Fig. 7 shows the reboiler liquid mole fraction of the component A.

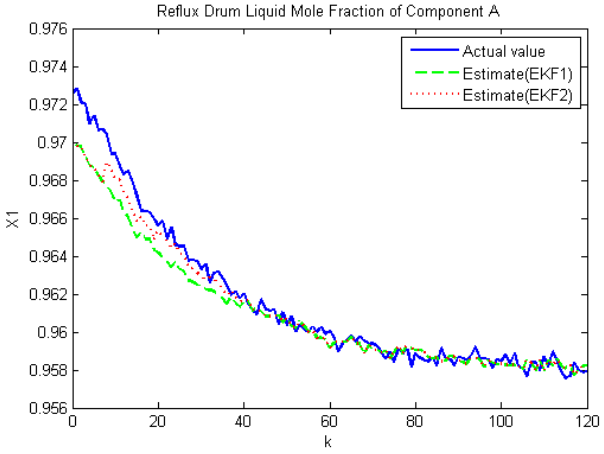

Fig. 6. Reflux drum liquid mole fraction of the component A.

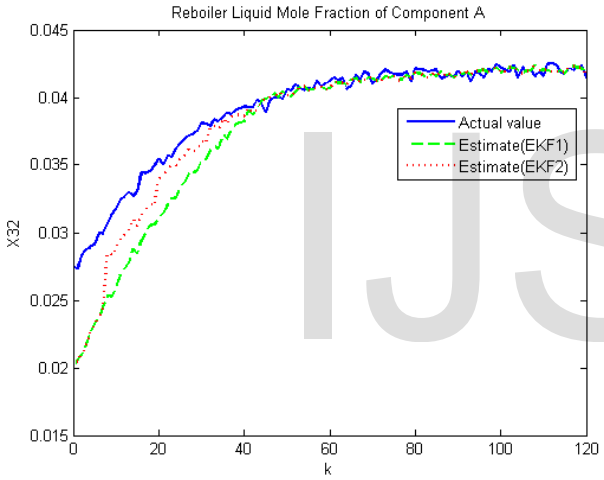

Fig. 7. Reboiler liquid mole fraction of the component A.

The mean square error (MSE), as defined by the following equation, is used to quantify the difference between the estimated profile in each of the EKF1 and EKF2 methods and its corresponding actual profile.

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{X}_i - X_i\right)^2 \quad (44)$$

where n is the number of sample points, $\hat{X}_i$ is the estimated value and X_i is the actual value. Table 3 gives the MSEs for the liquid mole fractions of the component A for both of the reflux drum and the reboiler.

These results indicate that for the distillation column case study, even in the presence of noise, each of the EKF2 estimators tracks the plant quite well whereas the estimators based only on the temperature measurements (EKF1) show significant offsets from their corresponding actual plant profiles.

TABLE 3
MEAN-SQUARED ERROR (IN UNITS OF 10^{-5})

Distillation column	Reflux mole fraction	Reboiler mole fraction
the EKF1 method (only secondary measurements)	0.1089	0.6525
MSE of the EKF2 method (secondary and delayed primary measurements)	0.0631	0.3727

7.2 Simulation Results of the CSTR

Details of the mathematical model derivation for the CSTR are given in section 6.2. The process is initially at an unsteady state and the estimators are given an incorrect starting estimate. Both the EKF1 and EKF2 methods are considered. The performances of the estimators are tested in the presence of the state and measurement noises. The noise variance used is as follows. The measurement error covariance for the temperature of reaction mixture $R^1 = 10^{-3}$ and for the delayed concentration of the component A, $R^2 = 10^{-6}$.

In the EKF1 method, the secondary temperature measurements are available at every sampling instant. In the EKF2 method, the samples for the primary composition are taken at an irregular interval that varies between 6-10 min, starting from the time step of k=12. The delay T_D in the measurement arrival is equal to 4 min.

Fig. 8 shows the simulation and estimation results for the CSTR. In this figure, the solid line represents the plant (actual values), the dashed line represents the EKF1 method estimation response and the doted line represents the EKF2 method estimation response.

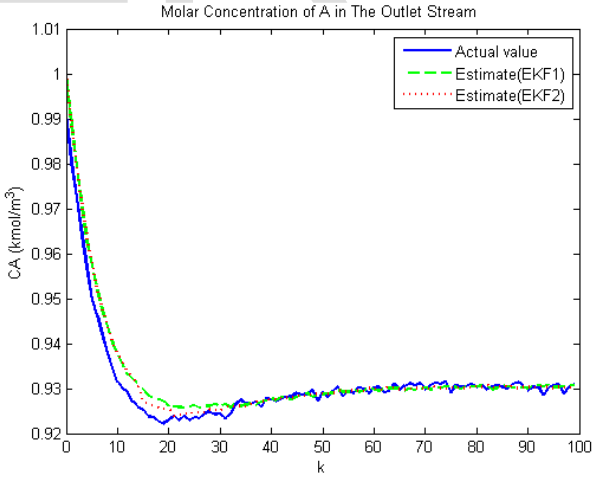

Fig. 8. Molar concentration of A in the outlet stream.

Table 4 gives the MSE for the concentration of A in the outlet stream. The MSEs are calculated for the two estimation methods (EKF1 and EKF2) discussed in this work.

These results indicate that for the CSTR case study even in the presence of noise, the EKF2 estimator tracks the plant quite well whereas the estimator based only on the temperature measurements (EKF1) shows significant offsets from the

actual plant.

TABLE 4
MEAN-SQUARED ERROR (IN UNITS OF 10^{-4})

Continuous stirred tank reactor	Concentration of A in the outlet stream
the EKF1 method (only secondary measurements)	0.8154
MSE of the EKF2 method (secondary and delayed primary measurements)	0.7096

8 CONCLUSION

In this work, the extended Kalman filter is employed for sensor data fusion. The estimation of states is carried out in the presence of noise. The data fusion method has been proposed to deal with multi-rate sensors systems. The fused estimate is better than the Kalman filtering result based on each single sensor's information. The EKF was employed to fuse primary and secondary measurement data in a distillation column and a CSTR. In the presented case studies, the estimators performed significantly better than the estimators exclusively employing the secondary measurements. The method presented in this work can be employed to deal with unknown (and arbitrarily varying) delays in the primary process variables. In addition, it is applicable to other estimation techniques and other filters, such as UKF, MHE or particle filters.

REFERENCES

[1] P. Kadlec, B. Gabrys, and S. Strandt, "Data-Driven Soft Sensors in the Process Industry," *Computers and Chemical Engineering*, vol. 33, no. 4, pp. 795-814, 2009.

[2] D. L. Hall, and J. Llinas, *Multisensor data fusion*. CRC press, 2001.

[3] A. Jahanmiri and H. Rasooli, "On-Line States and Parameter Identification of Acetone–Butanol–Ethanol Fermentation Process," *Biochemical engineering journal*, vol. 24, no. 2, pp. 115-123, 2005.

[4] T.D. Larsen, N.A. Andersen, O. Ravn, and N.K. Poulsen, "Incorporation of Time Delayed Measurements in a Discrete-Time Kalman Filter," *Proc. IEEE In Decision and Control('37)*, pp. 3972-3977, Dec. 1998.

[5] R.D. Gudi, S.L. Shah, and M.R. Gray, "Multirate State and Parameter Estimation in an Antibiotic Fermentation with Delayed Measurements," *Biotechnology and Bioengineering*, vol. 44, no. 11, pp. 1271-1278, 1994.

[6] S. Tatiraju, M. Soroush, and B.A. Ogunnaike, "Multirate Nonlinear State Estimation with Application to a Polymerization Reactor," *AIChE Journal*, vol. 45, no. 4, pp. 769-780, 1999.

[7] R.K. Mutha, W.R. Cluett, and A. Penlidis, "A new Multirate-Measurement-based Estimator: Emulsion Copolymerization Batch Reactor Case Study," *Industrial and Engineering Chemistry Research*, vol. 36, no. 4, pp. 1036-1047, 1997.

[8] R. Amirthalingam, S.W. Sung, and J.H. Lee, "Two-Step Procedure for Data-Based Modeling for Inferential Control Applications," *AIChE journal*, vol. 46, no. 10, pp. 1974-1988, 2000.

[9] C.V. Rao, J.B Rawlings, and D.Q. Mayne, "Constrained State Estimation for Nonlinear Discrete-Time Systems: Stability and Moving Horizon Approximations," *IEEE Tranc. Automatic Control*, vol. 48, no. 2, pp. 246-258, 2003.

[10] P. Vachhani, R. Rengaswamy, V. Gangwal, and S. Narasimhan, "Estimation in Constrained Nonlinear Dynamical Systems," *AIChE Journal*, vol. 51, no. 3, pp. 946-959, 2005.

[11] A. Gopalakrishnan, N.S. Kaisare, and S. Narasimhan, "Incorporating Delayed and Infrequent Measurements in Extended Kalman Filter Based Nonlinear State Estimation," *Journal of Process Control*, vol. 21, no. 1, pp. 119-129 , 2011.

[12] K. Berntorp, K.E. Arzén, and A. Robertsson, "Sensor Fusion for Motion Estimation of Mobile Robots with Compensation for Out-Of-Sequence Measurements," *Proc. IEEE In Control, Automation and Systems (ICCAS '11)*, pp. 211-216, Oct. 2011.

[13] J.B. Gao, and C.J. Harris, "Some Remarks on Kalman Filters for the Multisensor Fusion," *Information Fusion*, vol.3, no. 3, pp. 191-201, 2002.

[14] L. Hong, S. Cong, and D. Wicker, "Multirate Interacting Multiple Model (MRIMM) Filtering with Out-Of-Sequence GMTI Data," IEE Proc. Radar, Sonar, Navigation, vol. 150, no. 5, pp. 333-343, 2003.

[15] J. Hahn, and T.F. Edgar, "An Improved Method for Nonlinear Model Reduction Using Balancing of Empirical Gramians," Computers & chemical engineering, vol. 26, no. 10, pp. 1379-1397, 2002.